Sub Sequence and sequential limit

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I. SUB-SEQUENCE

Let $\{x_n\}$ be a sequence of real numbers. Let us consider a sequence which consists terms of the above sequence as $\{x_2, x_5, x_8, x_{11}, ...x_{n_k}.....\}$,it is noted as $\{x_{n_k}\}_{n\epsilon N}$. This is called sub-sequence of $\{x_n\}$, So we can form infinitely many sub-sequence from the given sequence.

A. Examples

1.$x_n = \sin(\frac{n\pi}{2})$; then we can form the sub-sequences $\{1, -1, 1, -1, ...\}$,$\{1, 0, 1, 0.......\}$ etc.
2.$x_n = (-1)^n$; we can form sub-sequences $\{1, 1, 1, ...\}$ and $\{-1, -1, -1.....\}$,clearly these sub-sequences are convergent, whereas $\{x_n\}$ is not.
Theorem 1: If a sequence is convergent to l then each of its sub-sequence are convergent to l.
But the convergent of the sub-sequence's does not imply the convergence of sequence, as the above example(2) shows.
Theorem 2: If all the sub-sequences of a sequence convergent to the same limit,then $\{x_n\}$ is convergent.
Bolzano-weierstrass theorem : Every bounded sequence of real numbers has a convergent sub-sequence.
Worl out:1. Justify converse of Bolzano theorem .
2.Fine the possible convergent sub-sequences of $x_n = \cos(\frac{n\pi}{6})$.
3. Prove that if $\{x_{2n-1}\}$ and $\{x_{2n}\}$ convergent to l, then $\{x_n\}$ converges l.
4.Define cluster point.
5.Prove that $\lim(1 + \frac{1}{2n})^n = \sqrt{e}$.

B. upper and lower limit

The greatest sub-sequential limit of a sequence $\{x_n\}$ is called upper limit of limit superior and is denoted as $\limsup x_n$, and the least sub-sequential limit is called lower limit or limit inferior denoted $\liminf x_n$.

Examples:

1. Find the upper and lower limit of $x_n = (-1)^n(1 + \frac{1}{n})$.
2.$x_n = (-1)^n n^2$, show $\limsup x_n = \infty$ $\liminf x_n = -\infty$.

Cauchy Limit theorem **First Limit Theorem:** If $\lim x_n = l$ then $\lim \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = l$
Corollary:$\sqrt[n]{x_1x_2x_3.....x_n} = l$

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Work out:1.Show $\lim \frac{1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{n}}{n}=0$.

2.$\lim \frac{1+\sqrt{2}+^3\sqrt{3}+.....^n\sqrt{n}}{n}=1$

3.Show that a sequence is convergent iff$\overline{\lim} x_n = \underline{\lim} x_n$.

Second limit theorem: If $x_n > 0$, and $\lim \frac{x_{n+1}}{x_n} = l$ iff $^n\sqrt{x_n} = l$

1.Prove that $\lim \frac{(n!)^{\frac{1}{n}}}{n} = \frac{1}{e}$.

2.$\lim\{(2n+1)(2n+2)(2n+3)......(2n+n)\}^{\frac{1}{n}} = \frac{27}{4e}$.