Logic-II

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I. PRECEDENCE OF LOGICAL OPERATORS.

Like arithmetic operators, logical operators have precedence that determines how things are grouped in the absence of parentheses. In an expression, the operator with the highest precedence is grouped with its operand(s) first, then the next highest operator will be grouped with its operands, and so on. This we have to keep in mind when we use logical operations.

A. Formal logic

The abstract study of propositions, statements, or assertively used sentences and of deductive arguments. The discipline abstracts from the content of these elements the structures or logical forms that they provide.
A formal system is an organization of terms used for the analysis of deduction. It consists of an alphabet, a language over the alphabet to construct sentences, and a rule for deriving sentences. Among the important properties that logical systems can have are:
Consistency- Which means that no theorem of the system contradicts another. **Validity-** Which means that the system's rules of proof never allow a false inference from true premises. **Completeness-** Which means that if a formula is true, it can be proven, i.e. is a theorem of the system. **Soundness-** Meaning that if any formula is a theorem of the system, it is true. This is the converse of completeness.
(Note that in a distinct philosophical use of the term, an argument is sound when it is both valid and its premises are true) **Expressivity-** Meaning what concepts can be expressed in the system.

Tautologies

A truth function of n arguments is defined to be a function of n arguments, the arguments and values of which are the truth values T or F. As we have seen, any statement form containing n distinct statement letters determines a corresponding truth function of n arguments.
A statement form that is always true, no matter what the truth values of its statement letters may be, is called a **tautology**. A statement form is a tautology if and only if its corresponding truth function takes only the value T, or equivalently, if, in its truth table, the column under the statement form contains only Ts.
Examples: An example of a tautology is $(A \vee (\neg A))$,the so-called law of the excluded middle. Other simple examples are $(\neg(A \wedge (\neg A))), (A \Leftrightarrow (\neg(\neg A))), ((A \wedge B) \Rightarrow A), and (A \Rightarrow (A \vee B))$.
B is said to logically imply C (or, synonymously, C is a **logical consequence** of B) if and only if every truth assignment to the statement letters of B and C that makes B true also makes C true. For example, $(A \wedge B)$logically implies A, A logically implies $(A \vee B)$, and$(A \wedge A \Rightarrow B))$ logically implies B.
Proposition 1.1 a. B logically implies C if and only if $(B \Rightarrow C)$ is a tautology. b. B and C are logically equivalent if and only if$(B \Leftrightarrow C)$ is a tautology. Proof a. (i) Assume B logically implies C. Hence, every truth assignment that makes B true also makes C true. Thus, no truth assignment makes B true and C false. Therefore, no truth assignment makes $(B \Rightarrow C)$ false, that is, every truth assignment makes $(B \Rightarrow C)$ true.
In other words, $(B \Rightarrow C)$is a tautology. (ii) Assume $(B \Rightarrow C)$ is a tautology. Then, for every truth assignment, $(B \Rightarrow C)$ is true, and, therefore, it is not the case that B is true and C false. Hence, every truth assignment that

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makes B true makes C true, that is, B logically implies C. b.$(B \Leftrightarrow C)$ is a tautology if and only if every truth assignment makes $(B \Leftrightarrow C)$ true, which is equivalent to saying that every truth assignment gives B and C the same truth value, that is, B and C are logically equivalent.
Examples: determine whether $A \Leftrightarrow ((\neg B) \vee C)) \Rightarrow (\neg A) \Rightarrow B)$ is a tautology.
Every one is advice to read Mendelson Chapter 1 and 2.